Taming the Leverage Cycle

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An anecdote about a leverage cycle

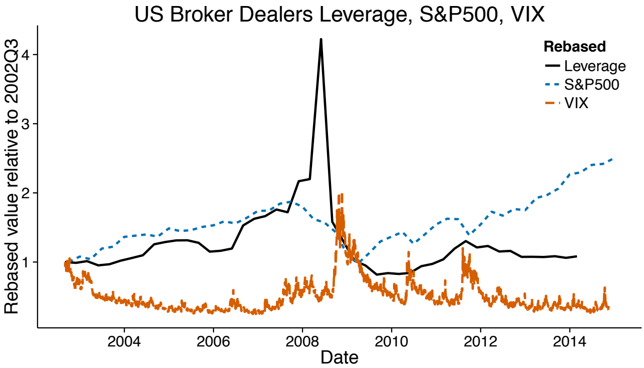

Figure: Leverage of US Broker-Dealers (solid black line), S&P500 index (dashed blue line), VIX S&P500 (red dash-dotted line.

Strong co-movement: Can we connect these variables in a simple dynamic model?

Prior work on leverage cycles

Important contributions

- Geanakoplos, 2003 and 2010 $\rightarrow$ leverage cycles in rational 2 period model;
- Adrian and Shin, 2008 $\rightarrow$ empirical study of procyclical leverage;
- Poledna et al., 2013 $\rightarrow$ leverage and heavy tailed returns.

Main ideas in summary

- Banks use leverage (Assets/Equity) to boost returns
- Ability to leverage depends on market risk
- If risk is low leverage is high, if risk is high leverage is low
- Leveraging up pushed prices up, deleveraging pushes prices down

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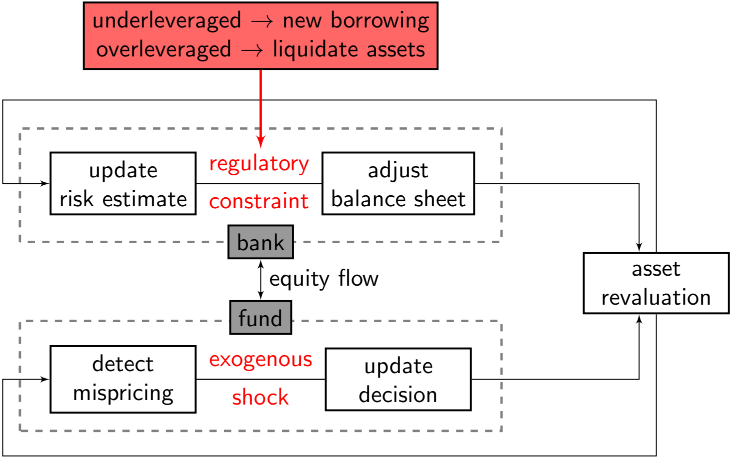
Our aim: study this in dynamical system of the "form"

$$\begin{aligned}\text{Leverage} &= F\left(\text{Perceived risk}\right),\\ \text{Prices} &= G\left(\text{Leverage}\right),\\ \text{Perceived risk} &= H\left(\text{Prices}\right).\end{aligned}$$

Stochastic discrete time model of leverage cycles



Outline for the remainder of this talk

1. A model of a leveraged bank and a fund investor.

2. Emergence of endogenous risk $\rightarrow$ leverage cycles.

3. Optimal leverage policy in the presence of both exogenous and endogenous risk.

Bank leverage regulation

Motivation: VaR constraint with normal log returns

$$\lambda(t) \leq \bar{\lambda}(t) = F_{\mathrm{VaR}}(\sigma^2(t)) = \frac{1}{\sigma(t)\Phi^{-1}(a)} \propto \frac{1}{\sigma(t)}.$$

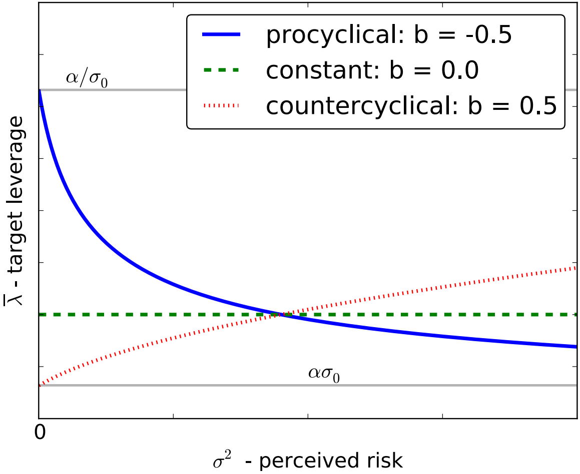
Our model: 3 parameter leverage constraint

$$\lambda(t) \leq \bar{\lambda}(t) = F_{(\alpha,\sigma_0^2,b)}(\sigma(t)) := \alpha(\sigma^2(t) + \sigma_0^2)^b.$$

Note:

- Due to profit maximization: $\lambda(t) \approx \bar{\lambda}(t) :=$ target leverage,
- α: bank risk level (leverage at a given level of risk),
- $b < 0$: procyclical w.r.t $\sigma(t)$,
- $b > 0$: countercyclical w.r.t $\sigma(t)$,
- σ_0: lower/upper bound on leverage.

Cyclicality parameter b: procyclical vs. countercyclical policies



For now focus on Value-at-Risk ($b = -0.5$) only.

Risk estimation and portfolio adjustment

Historical estimation of volatility

Let $p(t)$ be the price of the risky asset at time t. Then the bank's **perceived risk** evolves as

$$\sigma^2(t+\tau) = (1-\tau\delta)\sigma^2(t) + \tau\delta\left(\log\left[\frac{p(t)}{p(t-\tau)}\right]\frac{t_{\mathrm{VaR}}}{\tau}\right)^2.$$

Balance sheet

Adjust size of balance sheet to meet target leverage:

$$\Delta B(t) = \tau\theta\{\overline{\lambda}(t)(A_{\mathrm{B}}(t) - L_{\mathrm{B}}(t)) - A_{\mathrm{B}}(t)\}.$$

Adjust equity to meet equity target:

$$\kappa_{\mathrm{B}}(t) = \tau\eta\{\overline{E} - (A_{\mathrm{B}}(t) - L_{\mathrm{B}}(t))\}$$

The fund stabilizes the price dynamics of the risky asset

Fund characteristics:

- Not leveraged.
- Fund has a notion of a fundamental value μ of the risky asset.
- Dynamics of portfolio weight for risky asset:

$$\Delta w_F(t+\tau) \propto \rho(\mu - p(t)) + \sqrt{\tau} s(t) \xi(t),$$

where $\xi(t) \sim \mathcal{N}(0,1)$ and $s(t)$ follow GARCH(1,1).

Note:

- Fund stabilizes prices (buys if price below fundamental, sells above).
- For $s = 0$ we obtain deterministic system.
- Fund is source of “clustered” exogenous volatility.

Market mechanism for risky asset

1. Bank and fund demand function:

$$D_{\mathrm{B}}(t+\tau) = \frac{1}{p(t+\tau)} w_{\mathrm{B}}(n(t)p(t+\tau) + c_{\mathrm{B}}(t) + \Delta B(t)),$$
$$D_{\mathrm{F}}(t+\tau) = \frac{1}{p(t+\tau)} w_{\mathrm{F}}(t+\tau)((1-n(t))p(t+\tau) + c_{\mathrm{F}}(t)).$$

2. Compute $p(t+\tau)$ by market clearing:

$$1 = D_{\mathrm{B}}(t+\tau) + D_{\mathrm{F}}(t+\tau)$$

3. Compute new ownership of risky asset for bank $n(t+\tau)$ and fund $1-n(t+\tau)$

We can collect full model in 6D map

Map:

$$x(t+\tau) = g(x(t))$$

State vector:

$$x(t) = [p(t), \sigma^2(t), n(t), L_{\mathrm{B}}(t), w_F(t), p'(t)]^T,$$

where:

- p: Price of risky asset.
- σ^2: Perceived risk.
- n: Amount of asset owned by bank.
- L_{B}: Liabilities of bank.
- w_F: Investment into risky asset by fund.
- p': Past price of risky asset.

Examples of leverage cycles: we consider four parameter scenarios

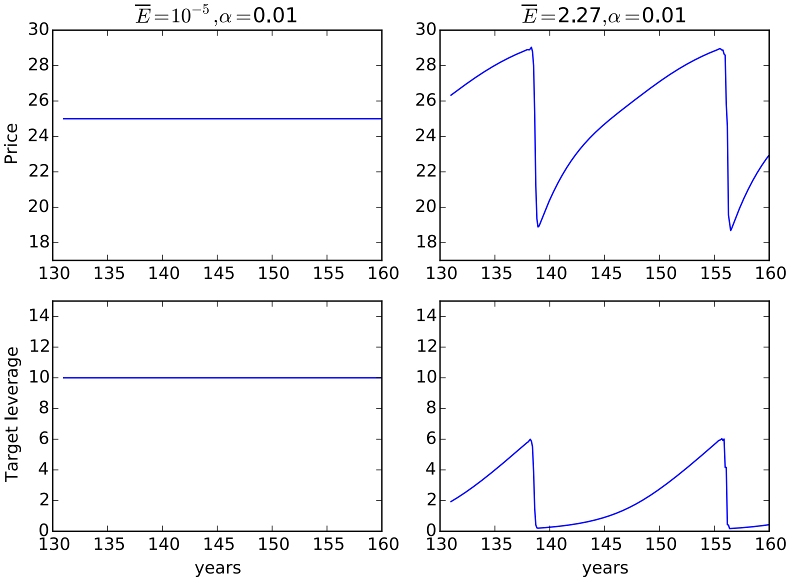
(i) Deterministic, small bank (weak endogenous risk): $\overline{E} = 10^{-5}$ and $s = 0$,

(ii) Deterministic, large bank (strong endogenous risk): $\overline{E} = 2.27$ and $s = 0$,

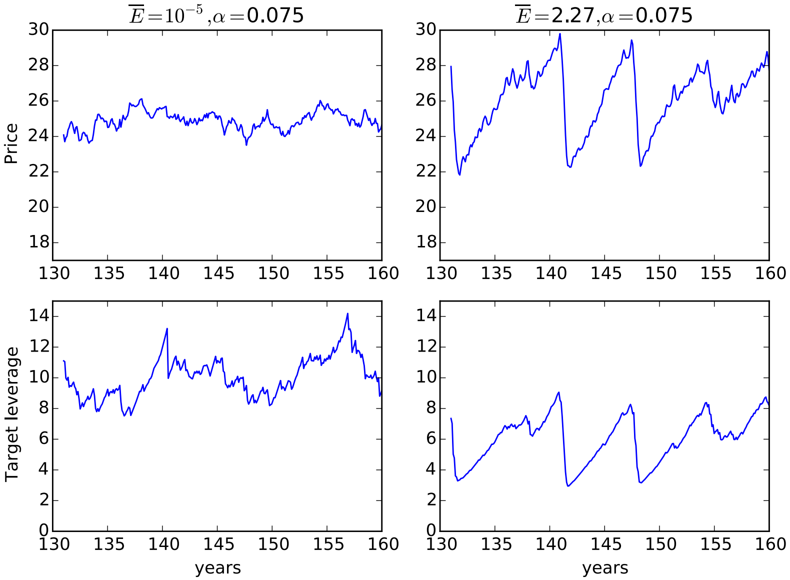
(iii) Stochastic, small bank (weak endogenous risk): $\overline{E} = 10^{-5}$ and $s > 0$.

(iv) Stochastic, large bank (strong endogenous risk): $\overline{E} = 2.27$ and $s > 0$,

Deterministic: (i) small bank vs. (ii) large bank



Stochastic: (iii) small bank vs. (iv) large bank



How do leverage cycles depend on the model parameters?

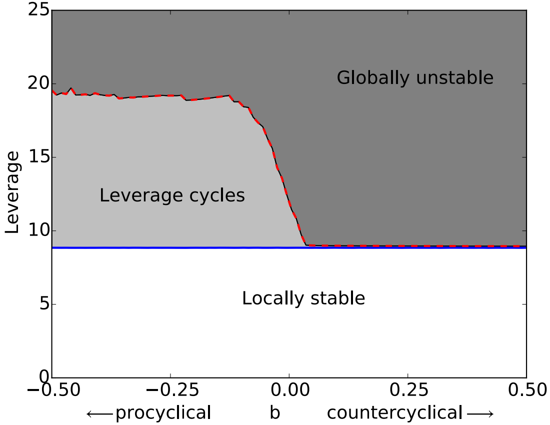


Figure: Deterministic model (eigenvalues)

Leverage cycles only in procyclical region.

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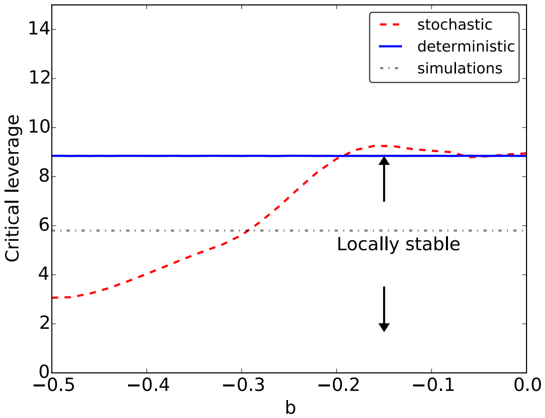


Figure: Critical leverage for emergence of leverage cycles: deterministic/stochastic (Lyapunov exponents)

Stochastic model destabilizes for smaller levels of leverage.

How do leverage cycles depend on the model parameters?

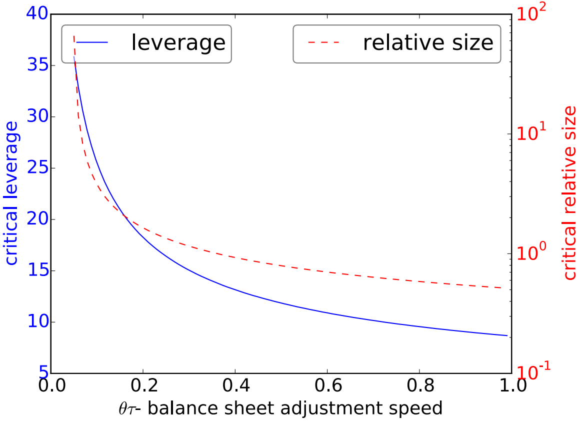


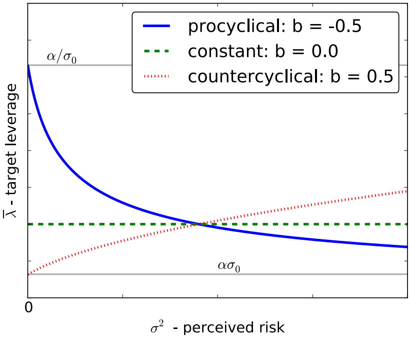
Figure: Critical leverage as a function of balance sheet adjustment speed

Slower balance sheet adjustment stabilizes the system.

Reminder: bank leverage policies

Target leverage

$$\lambda(t) \leq \bar{\lambda}(t) = F_{(\alpha,\sigma_0^2,b)}(\sigma(t)) := \alpha(\sigma^2(t) + \sigma_0^2)^b.$$



How do different values of "b" affect the overall volatility in the system?

Intuition: Endogenous vs. exogenous volatility

Risk management dilemma

- Microprudential: Should reduce leverage when **exogenous** volatility is high.
- Macroprudential: Leverage adjustment can lead to even higher **endogenous**.

Intuition for our model

- Small bank + strong exogenous volatility:
  Value-at-Risk is optimal
  ($b = -0.5$)
- Large bank + low exogenous volatility:
  Constant leverage is optimal
  ($b = 0$)

What is the right trade off between micro- and macroprudential perspective?

Optimal cyclicality? – it depends

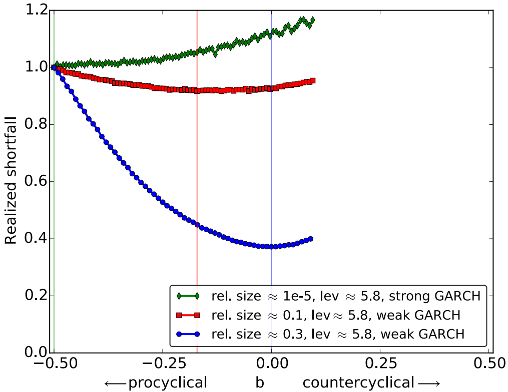


Figure: Realized shortfall (average large losses of bank) at constant leverage.

Optimal cyclicality crucially depends on bank size and strength of exogenous volatility.

Conclusions

1. Endogenous amplification of exogenous shocks as unintended consequence of regulation.
2. Crucial determinants of endogenous volatility:
   - Leverage and size of leveraged investor
   - Balance sheet adjustment speed: Dynamics and timescales matter!
3. Better leverage policies?
   i Value-at-Risk is optimal if leveraged investor is small and lots of exogenous volatility
   ii Constant leverage is optimal if leveraged investors is large and little exogenous volatility

Open question: which regime (i or ii) do we live in?

BACK UP

Full 6 D model (1/2)

Recall:

$$x(t) = [\sigma^2(t), w_\mathrm{F}(t), p(t), n(t), L_\mathrm{B}(t), p'(t)]^\mathrm{T}, \tag{1}$$

Definitions:

$$\begin{aligned}
&\text{Bank assets} & A_\mathrm{B}(t) &= p(t)n(t)/w_\mathrm{B}, \\
&\text{Target leverage} & \bar{\lambda}(t) &= \alpha(\sigma^2(t) + \sigma_0^2)^b, \\
&\text{Balance sheet adjustment} & \Delta B(t) &= \tau\theta(\bar{\lambda}(t)(A_\mathrm{B}(t) - L_\mathrm{B}(t)) - A_\mathrm{B}(t)), \\
&\text{Equity redistribution} & \kappa_\mathrm{B}(t) &= -\kappa_\mathrm{F}(t) = \tau\eta(\overline{E} - (A_\mathrm{B}(t) - L_\mathrm{B}(t))), \\
&\text{Bank cash} & c_\mathrm{B}(t) &= (1 - w_\mathrm{B})n(t)p(t)/w_\mathrm{B} + \kappa_\mathrm{B}(t), \\
&\text{Fund cash} & c_\mathrm{F}(t) &= (1 - w_\mathrm{F}(t))(1 - n(t))p(t)/w_\mathrm{F}(t) + \kappa_\mathrm{F}(t).
\end{aligned} \tag{2}$$

Full 6 D model (2/2)

Dynamical system:

$$x(t+\tau) = g(x(t)) \tag{3}$$

where the function g is the following 6-dimensional map:

$$\sigma^2(t+\tau) = (1-\tau\delta)\sigma^2(t) + \tau\delta\left(\log\left[\frac{p(t)}{p'(t)}\right]\frac{t_{\mathrm{VaR}}}{\tau}\right)^2, \tag{4a}$$

$$w_{\mathrm{F}}(t+\tau) = w_{\mathrm{F}}(t) + \frac{w_{\mathrm{F}}(t)}{p(t)}\left[\tau\rho(\mu - p(t)) + \sqrt{\tau}s\xi(t)\right], \tag{4b}$$

$$p(t+\tau) = \frac{w_{\mathrm{B}}(c_{\mathrm{B}}(t) + \Delta B(t)) + w_{\mathrm{F}}(t+\tau)c_{\mathrm{F}}(t)}{1 - w_{\mathrm{B}}n(t) - (1-n(t))w_{\mathrm{F}}(t+\tau)}, \tag{4c}$$

$$n(t+\tau) = \frac{w_{\mathrm{B}}(n(t)p(t+\tau) + c_{\mathrm{B}}(t) + \Delta B(t))}{p(t+\tau)}, \tag{4d}$$

$$L_{\mathrm{B}}(t+\tau) = L_{\mathrm{B}}(t) + \Delta B(t), \tag{4e}$$

$$p'(t+\tau) = p(t). \tag{4f}$$

Back

Guiding principles for choice of main parameters

1. Properties of the leverage cycle:
   - Peak-to-trough ratio ≈ 2,
   - Period of cycles ≈ 10 years,

determines α (bank risk level), $\bar{E}$ (bank equity target).

2. Timescale for risk estimation:
   - $t_\delta = 1/\delta \approx 2$ years (based on RiskMetrics).